

## Operations and Algebraic Thinking, Grades K-2

Kindergarten	Grade 1	Grade 2
<p><b>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</b></p> <ol style="list-style-type: none"> <li>1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</li> <li>2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.</li> <li>3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., <math>5 = 2 + 3</math> and <math>5 = 4 + 1</math>).</li> <li>4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.</li> <li>5. Fluently add and subtract within 5.</li> </ol>	<p><b>Represent and solve problems involving addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</li> <li>2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</li> </ol> <p><b>Understand and apply properties of operations and the relationship between addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>3. Apply properties of operations as strategies to add and subtract. Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition.) To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.)</li> <li>4. Understand subtraction as an unknown-addend problem. For example, subtract <math>10 - 8</math> by finding the number that makes 10 when added to 8.</li> </ol> <p><b>Add and subtract within 20.</b></p> <ol style="list-style-type: none"> <li>5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</li> <li>6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., <math>8 + 6 = 8 + 2 + 4 = 10 + 4 = 14</math>); decomposing a number leading to a ten (e.g., <math>13 - 4 = 13 - 3 - 1 = 10 - 1 = 9</math>); using the relationship between addition and subtraction (e.g., knowing that <math>8 + 4 = 12</math>, one knows <math>12 - 8 = 4</math>); and creating equivalent but easier or known sums (e.g., adding <math>6 + 7</math> by creating the known equivalent <math>6 + 6 + 1 = 12 + 1 = 13</math>).</li> </ol> <p><b>Work with addition and subtraction equations.</b></p> <ol style="list-style-type: none"> <li>7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? <math>6 = 6</math>, <math>7 = 8 - 1</math>, <math>5 + 2 = 2 + 5</math>, <math>4 + 1 = 5 + 2</math>.</li> <li>8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations <math>8 + ? = 11</math>, <math>5 = \square - 3</math>, <math>6 + 6 = \square</math>.</li> </ol>	<p><b>Represent and solve problems involving addition and subtraction.</b></p> <ol style="list-style-type: none"> <li>1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</li> </ol> <p><b>Add and subtract within 20.</b></p> <ol style="list-style-type: none"> <li>2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</li> </ol> <p><b>Work with equal groups of objects to gain foundations for multiplication.</b></p> <ol style="list-style-type: none"> <li>3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</li> <li>4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</li> </ol>

## Operations and Algebraic Thinking, Grades 3-5

Grade 3	Grade 4	Grade 5
<p><b>Represent and solve problems involving multiplication and division.</b></p> <ol style="list-style-type: none"> <li>Interpret products of whole numbers, e.g., interpret <math>5 \times 7</math> as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as <math>5 \times 7</math>.</i></li> <li>Interpret whole-number quotients of whole numbers, e.g., interpret <math>56 \div 8</math> as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as <math>56 \div 8</math>.</i></li> <li>Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</li> <li>Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations <math>8 \times ? = 48</math>, <math>5 = \div 3</math>, <math>6 \times 6 = ?</math>.</li> </ol> <p><b>Understand properties of multiplication and the relationship between multiplication and division.</b></p> <ol style="list-style-type: none"> <li>Apply properties of operations as strategies to multiply and divide. <i>Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative property of multiplication.) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property.)</i></li> <li>Understand division as an unknown-factor problem. <i>For example, find <math>32 \div 8</math> by finding the number that makes 32 when multiplied by 8.</i></li> </ol> <p><b>Multiply and divide within 100.</b></p> <ol style="list-style-type: none"> <li>Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</li> </ol> <p><b>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</b></p> <ol style="list-style-type: none"> <li>Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</li> <li>Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></li> </ol>	<p><b>Use the four operations with whole numbers to solve problems.</b></p> <ol style="list-style-type: none"> <li>Interpret a multiplication equation as a comparison, e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</li> <li>Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</li> <li>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</li> </ol> <p><b>Gain familiarity with factors and multiples.</b></p> <ol style="list-style-type: none"> <li>Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</li> </ol> <p><b>Generate and analyze patterns.</b></p> <ol style="list-style-type: none"> <li>Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></li> </ol>	<p><b>Write and interpret numerical expressions.</b></p> <ol style="list-style-type: none"> <li>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</li> <li>Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as <math>2 \times (8 + 7)</math>. Recognize that <math>3 \times (18932 + 921)</math> is three times as large as <math>18932 + 921</math>, without having to calculate the indicated sum or product.</i></li> </ol> <p><b>2.1 Express a whole number in the range 2–50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as <math>2 \times 2 \times 2 \times 3</math>. CA</b></p> <p><b>Analyze patterns and relationships.</b></p> <ol style="list-style-type: none"> <li>Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i></li> </ol>

## The Number System, Grades 6-8

Grade 6	Grade 7	Grade 8
<p><b>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</b></p> <p>1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>. (In general, <math>(a/b) \div (c/d) = ad/bc</math>.) How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>3/4</math>-cup servings are in <math>2/3</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>3/4</math> mi and area <math>1/2</math> square mi?</i></p> <p><b>Compute fluently with multi-digit numbers and find common factors and multiples.</b></p> <p>2. Fluently divide multi-digit numbers using the standard algorithm.</p> <p>3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p> <p>4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</i></p> <p><b>Apply and extend previous understandings of numbers to the system of rational numbers.</b></p> <p>5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p>6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p> <p>7. Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right.</i></p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math>.</i></p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</i></p> <p>d. Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than 30 dollars.</i></p> <p>8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>	<p><b>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</b></p> <p>1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p> <p>b. Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p> <p>2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as <math>(-1)(-1) = 1</math> and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If <math>p</math> and <math>q</math> are integers, then <math>-(p/q) = (-p)/q = p/(-q)</math>. Interpret quotients of rational numbers by describing real world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p> <p>3. Solve real-world and mathematical problems involving the four operations with rational numbers.</p>	<p><b>Know that there are numbers that are not rational, and approximate them by rational numbers.</b></p> <p>1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</p> <p>2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., <math>\pi^2</math>). <i>For example, by truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i></p>

# Expressions and Equations, Grades 6-8

Grade 6	Grade 7	Grade 8
<p><b>Apply and extend previous understandings of arithmetic to algebraic expressions.</b></p> <ol style="list-style-type: none"> <li>Write and evaluate numerical expressions involving whole-number exponents.</li> <li>Write, read, and evaluate expressions in which letters stand for numbers.               <ol style="list-style-type: none"> <li>Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation "Subtract y from 5" as <math>5 - y</math>.</i></li> <li>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression <math>2(8 + 7)</math> as a product of two factors; view <math>(8 + 7)</math> as both a single entity and a sum of two terms.</i></li> <li>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas <math>V = s^3</math> and <math>A = 6s^2</math> to find the volume and surface area of a cube with sides of length <math>s = \frac{1}{2}</math>.</i></li> </ol> </li> <li>Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression <math>3(2 + x)</math> to produce the equivalent expression <math>6 + 3x</math>; apply the distributive property to the expression <math>24x + 18y</math> to produce the equivalent expression <math>6(4x + 3y)</math>; apply properties of operations to <math>y + y + y</math> to produce the equivalent expression <math>3y</math>.</i></li> <li>Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions <math>y + y + y</math> and <math>3y</math> are equivalent because they name the same number regardless of which number <math>y</math> stands for.</i></li> </ol> <p><b>Reason about and solve one-variable equations and inequalities.</b></p> <ol style="list-style-type: none"> <li>Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</li> <li>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</li> <li>Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers.</li> <li>Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</li> </ol> <p><b>Represent and analyze quantitative relationships between dependent and independent variables.</b></p> <ol style="list-style-type: none"> <li>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</i></li> </ol>	<p><b>Use properties of operations to generate equivalent expressions.</b></p> <ol style="list-style-type: none"> <li>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</li> <li>Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, <math>a + 0.05a = 1.05a</math> means that "increase by 5%" is the same as "multiply by 1.05."</i></li> </ol> <p><b>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</b></p> <ol style="list-style-type: none"> <li>Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional <math>\frac{1}{10}</math> of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar <math>9\frac{3}{4}</math> inches long in the center of a door that is 27 <math>\frac{1}{2}</math> inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i></li> <li>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.       <ol style="list-style-type: none"> <li>Solve word problems leading to equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. <i>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</i></li> <li>Solve word problems leading to inequalities of the form <math>px + q &gt; r</math> or <math>px + q &lt; r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. <i>For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</i></li> </ol> </li> </ol>	<p><b>Work with radicals and integer exponents.</b></p> <ol style="list-style-type: none"> <li>Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, <math>3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}</math>.</i></li> <li>Use square root and cube root symbols to represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that <math>\sqrt{2}</math> is irrational.</li> <li>Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as <math>3 \times 10^8</math> and the population of the world as <math>7 \times 10^9</math>, and determine that the world population is more than 20 times larger.</i></li> <li>Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</li> </ol> <p><b>Understand the connection between proportional relationships, lines, and linear equations.</b></p> <ol style="list-style-type: none"> <li>Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i></li> <li>Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation <math>y = mx</math> for a line through the origin and the equation <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</li> </ol> <p><b>Analyze and solve linear equations and pairs of simultaneous linear equations.</b></p> <ol style="list-style-type: none"> <li>Solve linear equations in one variable.       <ol style="list-style-type: none"> <li>Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</li> <li>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</li> </ol> </li> <li>Analyze and solve pairs of simultaneous linear equations.       <ol style="list-style-type: none"> <li>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</li> <li>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, <math>3x + 2y = 5</math> and <math>3x + 2y = 6</math> have no solution because <math>3x + 2y</math> cannot simultaneously be 5 and 6.</i></li> <li>Solve real-world and mathematical problems leading to two linear equations in two variables. <i>For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</i></li> </ol> </li> </ol>